Implicit self-adjusting computation for CostIt

Internship Defense

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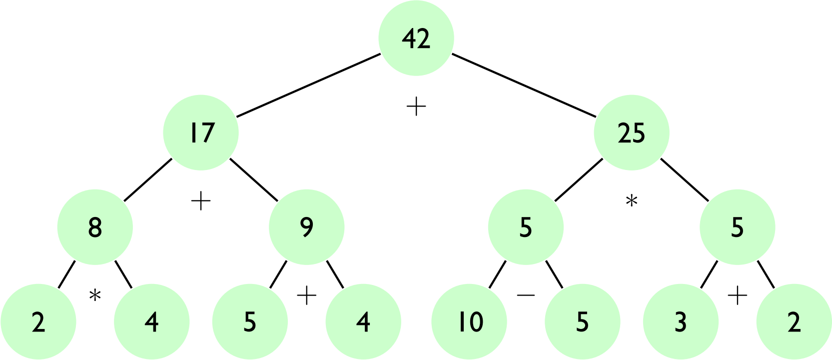
Self Adjusting Computation

- An evaluation mechanism that recomputes *only* the parts of the output that depend on inputs that have changed between runs
- *Change propagation* (CP) : the process of updating the parts of the output that depend on changed data
- *Implicit self-adjusting computation*: The program responds automatically to changes in its inputs without any manual effort from the programmer
- Often results in asymptotic speedup

Change Propagation by Example



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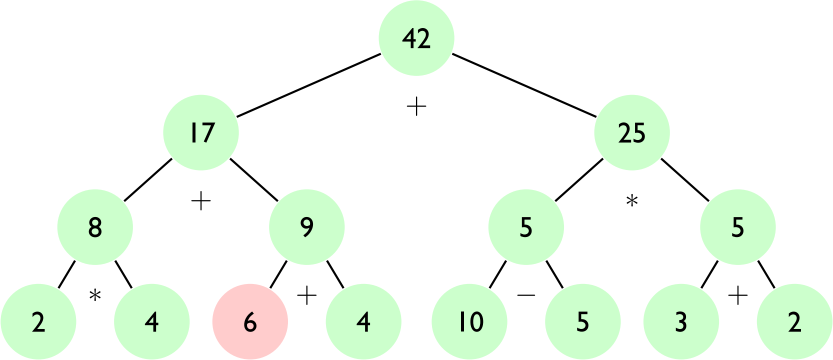

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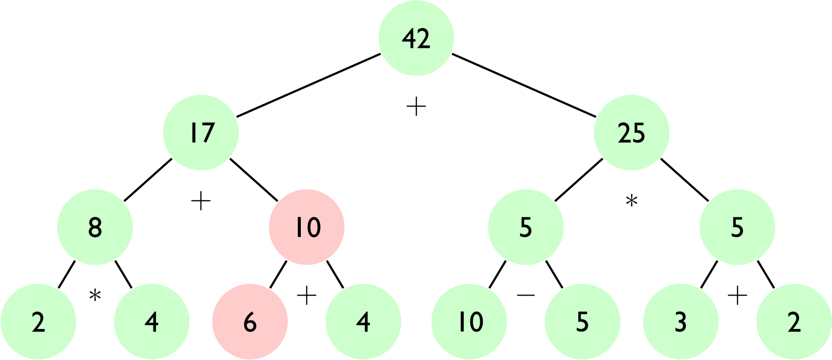

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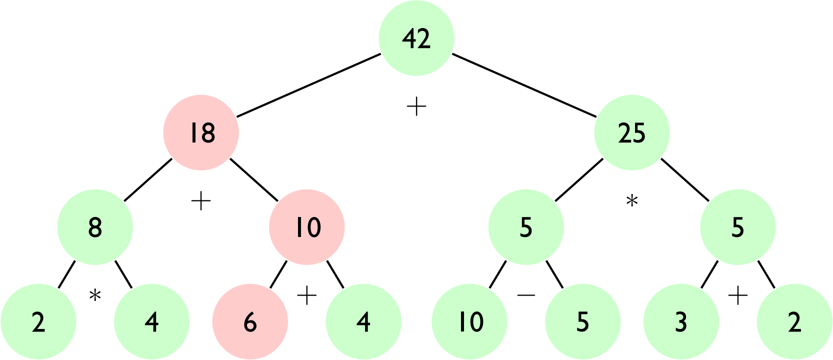

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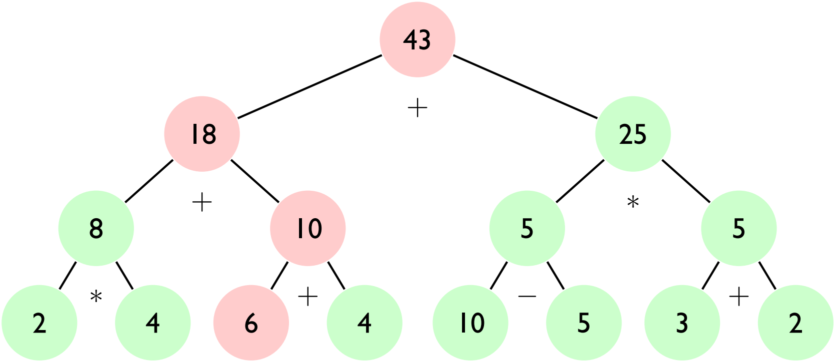

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- When $\epsilon = \mathbb{S}$ then κ is the upper bound of CP
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- Types have changeability annotations τ^{μ}
    - $\tau^{\mathbb{S}}$: a value that cannot change between runs
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- Index refinement types (in the style of DML)
- Lists : $\texttt{list}\,[n]^{\alpha}\ \tau$
  - A vector of n elements from which at most α can change

Running Example: `map` (typing)

$$\texttt{map} : (\tau_1 \xrightarrow{\mathbb{C}(\kappa)} \tau_2)^{\square} \xrightarrow{\mathbb{S}(0)} \texttt{list}\,[n]^{\alpha}\ \tau_1 \xrightarrow{\mathbb{S}(\kappa\cdot\alpha)} \texttt{list}\,[n]^{\alpha}\ \tau_2$$

- If `f` executes from-scratch with cost k and `l` has n elements of which at most α can change then `map f l` propagates changes with cost at most $\alpha \cdot \kappa$

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- If `f` executes from-scratch with cost k and `l` has n elements of which at most α can change then `map f l` propagates changes with cost at most $\alpha \cdot \kappa$
- Intuition: we need to recompute and update in place only the elements of the list that can change

Soundness (this internship)

- **Idea:** Translate a CostIt program to a self-adjusting program and show that the actual cost is no more that the cost derived by the type system

$$\begin{tikzcd}
\vdash^{\kappa}_{\epsilon} e : \tau \arrow[r, "\text{evaluates to}"] \arrow[d, "\text{translates to}"] & v_1 \arrow[d, phantom, "\approx"] \\
\vdash \ulcorner e \urcorner : \|\tau\| \arrow[r, "\text{evaluates in } c \text{ steps to}", "\text{if } \epsilon = \mathbb{C} \text{ then } c \leq \kappa"'] & v_2
\end{tikzcd}$$

(a) First run

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\end{tikzcd}$$

(b) Incremental run after input changes

Figure: Schematic representation of the basic properties of the translation

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$$\texttt{map} : (\texttt{int}^{\mathbb{C}} \xrightarrow{\mathbb{C}(\kappa)} \texttt{int}^{\mathbb{C}})^{\square} \xrightarrow{\mathbb{S}(0)} \texttt{list}\,[n]^{\alpha}\ \texttt{int}^{\mathbb{C}} \xrightarrow{\mathbb{S}(\kappa\cdot\alpha)} \texttt{list}\,[n]^{\alpha}\ \texttt{int}^{\mathbb{C}}$$

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$\quad \texttt{case}_{\mathrm{L}}\ e\ \texttt{of}$
$\quad \mid [\,]\ \to\ [\,]$
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    | h_r  →  let l  =  ref !(f h_r) in
              let ()  =
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- The translation is defined by induction on the typing derivation
- Two modes: $\epsilon = \mathbb{C}$ and $\epsilon = \mathbb{S}$
- The code generated in $\mathbb{C}$ mode will be executed from scratch during CP
- The code generated in $\mathbb{S}$ mode is self-adjusting
    - During this mode we record the computations that need to be re-executed during CP

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Similarity Relation

$$v_s \approx^{\tau}_{\sigma} v_t$$

- v_s is the source value, v_t is the target value
- σ is the store in the target
- Changeable values are references in the target (stored in σ)
- For *stable values*, v_s and v_t should coincide
- For *changeable values*, v_t should be a location and v_s should coincide with the value of this location in the store.

$$(3, 42) \approx^{\mathtt{int}^{\mathbb{S}} \times \mathtt{int}^{\mathbb{C}}}_{[l \mapsto 42]} (3, l)$$

Soundness, $\mathbb{C}$ mode

Theorem

Assume that

$$\vdash^{\kappa}_{\mathbb{C}} e : \tau \hookrightarrow \ulcorner e \urcorner$$

Then there exist v'_s, v'_t, σ', j *and* c, *such that*

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3. $\models c \mathrel{\dot{\leq}} \kappa$
4. $v'_s \approx^{\tau}_{\sigma'} v'_t$

Two-way similarity relation

$$(v_i,\ v_c) \approx^{\tau}_{(\sigma_i,\ \sigma_c)} v_t$$

- v_i is the initial source value, v_c is the source value after changes
- v_t is the target value that stores changeable values in references
- σ_i is the initial target store, σ_c is the target store holding changed values
- For *stable values*, v_i, v_c and v_t should coincide under the two stores
- For *changeable values*, v_i should be similar to v_t under σ_i and v_c should be similar to v_t under σ_c

$$((3, 42),\ (3, 43)) \approx^{\mathtt{int}^{\mathbb{S}} \times \mathtt{int}^{\mathbb{C}}}_{[l \mapsto 42],\ [l \mapsto 43]} (3, l)$$

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Then if $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* v_c', v_t', σ_f, σ_f', Q, j *and* c, *such that*

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Then if $[x\mapsto v_i]e\Downarrow v_i',\ j$ *then there exist* v_c'*,* v_t'*,* σ_f*,* σ_f'*,* Q*,* j *and* c*, such that*

(1) $[x\mapsto v_c]e\Downarrow v_c',\ j'$

(2) $[x\mapsto v_t]\ulcorner e\urcorner,\ \sigma_i\Downarrow v_t',\ \sigma_f,\ Q,\ c$

Soundness, $\mathbb{S}$ mode

Theorem

Assume that

$$\cdot;\cdot;x:\tau' \vdash^{\kappa}_{\mathbb{S}} e:\tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

Then if $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* v_c', v_t', σ_f, σ_f', Q, j *and* c, *such that*

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(3) $Q,\ \sigma_f[\sigma_c] \rightsquigarrow \sigma_f',\ c'$

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(3) $Q,\ \sigma_f[\sigma_c] \rightsquigarrow \sigma_f',\ c'$

(4) $\models c' \mathrel{\dot{\leq}} \kappa$

(5) $(v_i',\ v_c') \approx_{(\sigma_f,\ \sigma_f')} v_t'$

Proof Method

- The soundness is proved using logical relations
- We construct two Kripke step-indexed relational models
- Two fundamental properties, one for each typing mode
- The soundness theorems are corollaries of the fundamental properties of the logical relations

Summary

- Soundness proof for CostIt w.r.t. to concrete CP semantics
    - Older poof was w.r.t. an abstract semantics
- Designed a target language (saML) with infrastructure for CP
- Translated CostIt to saML
- Proved the correctness of the translation and the change propagation mechanism
- Proved that the cost derived by CostIt is a sound approximation of the actual cost (for both $\mathbb{C}$ and $\mathbb{S}$ modes)

Future Work

- Devise a more efficient CP mechanism
- Mechanize the proof using a proof assistant
- Adapt CostIt to derive the cost for demand-driven self-adjusting computation
- Ongoing work: Implementation of the type system using bidirectional type checking (E. Çiçek and D. Garg)

Thank You!

Questions?